GOLD VALUE WITH TRADABLE AND NON-TRADABLE GOODS IN A MULTI-COUNTRY GROWTH MODEL WITH FREE TRADE

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ABSTRACT

The purpose of this study is to examine gold price in global markets. We introduce gold into a general dynamic equilibrium growth model with multiple countries and free trades between countries. The model is developed by integrating the Solow growth model, the Uzawa two-sector growth model, and the Oniki–Uzawa trade model within a comprehensive framework. The model is built for any number of national economies and each national economy consists of one tradable and one non-tradable sectors. National economies are different in population, technologies, propensities to save, propensity to use gold, and propensities to consume. We show that the dynamics of the J-country world economy can be described by J differential equations. We simulate the model to demonstrate the existence of an equilibrium point, motion of the dynamic system, and (local) stability of the equilibrium point. We also demonstrate how changes in the propensities to use, the populations, the propensities to save, and the total factor productivities affect global economic development.

Keywords: trade pattern, gold value, tradable and non-tradable, economic growth, wealth accumulation

JEL: O4, F11

1. INTRODUCTION

It is almost trivial to mention the importance of studying dynamics of gold value in modern globalizing economies. Nevertheless, one might be surprised to know that economics still lacks a formal analytical (mathematical) framework to study how the price(s) of gold is determine in global market. The contemporary global free trade with instantaneous information flows without geographical barriers has made the motion of price gold a global concern. Nevertheless, dynamics of gold price has almost been neglected in the literature of economic dynamics with microeconomic foundation. The main reason for this lack of interest is that economics does not have a proper analytical framework to take account of the economic mechanisms of gold price determination within a general analytical framework with microeconomic foundation. The unique feature of this paper is introduction of gold into the neoclassical growth model. "What is gold? Is it money, currency, an investment or wealth? … Gold is whatever its users think it is. And … the vast majority of the above-ground gold, today somewhere around 165,000 tons, is held by people who understand it as wealth." (FOFOA, 2012). Gold plays the role of storing value. It is owned as a diversified share portfolio. It is also used for decoration and a sign of social status. It is held as a symbol of power and wealth. Possible multiple roles of gold in social and economic life explain partly why the value of gold has never been properly analyzed in economic theory. We emphasize dynamics of gold value as determinants of gold values are not like most other commodities. On the supply side the amount of gold stock changes but very slowly. On the demand side, most of gold is used

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for jewelry, coin collectors and central banks, with less than 10% used by industrial production (Thomas, 2015). There are some dynamic models on gold prices (Barro, 1979; Bordo and Ellson, 1985; Dowd and Sampson, 1993; Chappell and Dowd, 1997). As wealth accumulation and portfolio structure can be carried in form of holding gold and accumulating physical wealth, dynamics of gold price should be closely related to physical capital accumulation and other variables. Moreover, as national markets are increasingly integrated, gold price cannot be properly treated without taking account of international trade. We study dynamic interdependence between economic growth, structural change, and gold price in well-integrated global markets.

This study is primarily concerned with dynamics of gold value in an analytical framework with interactions among wealth and physical capital accumulation and trade patterns between multiple national economies. In our approach global economic growth is mainly enforced by physical capital accumulation. As gold may be held as wealth, the preference for using and holding gold will also make a contribution to economic growth and global trade pattern formation. The global growth mechanism of physical accumulation is based on the Solow growth model. We describe international trade patterns on the basis of the dynamic trade models with accumulating capital developed by Oniki and Uzawa and others (for instance, Oniki and Uzawa, 1965; Frenkel and Razin, 1987; Sorger, 2002; and Nishimura and Shimomura, 2002). The Oniki-Uzawa model is constructed for the two-country with two goods. We use tradable good and non-tradable good rather than capital goods and consumer goods as in the Oniki-Uzawa model. Distinction between tradable good and non-tradable good is significant for explaining the terms of trade (Mendoza, 1995; Stockman and Tesar, 1995), for explaining the exchange rate (Stulz, 1987; Stockman and Dellas, 1989; Backus and Smith, 1993; Rogoff, 2002); for dealing with current account dynamics (Edwards, 1989), or for solving the home premium puzzle (Baxter et al., 1998; Pesenti and van Wincoop, 2002). A reason for this distinction is given by Backus and Smith (1993:1) as follows: "The mechanism is fairly simple. Although the law of one price holds, in the sense that each good sells for a single price in all countries, PPP may not: price indexes combine prices of both traded and nontraded goods, and because the latter are sold in only one country their prices, and hence price indexes, may differ across countries." This paper introduces gold into the multi-country growth models with international trade and economic structure proposed by Zhang (2010, 2015). The analytical framework treats the global economy as an integrated whole. The economic system is built on the basis of the Solow model, the Uzawa two-sector model and the Oniki-Uzawa trade model. Different from the growth models with the Ramsey approach, we use the alternative utility function proposed by Zhang (1993) to determine saving and consumption. We analyze trade issues within the framework of a simple international macroeconomic growth model with perfect capital mobility. The rest of the paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the motion of the global economy. Section 4 carries out comparative dynamic analysis to examine the impact of changes in some parameters on the motion of the global economy. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. THE MODEL

The model in this study is developed within the framework of the neoclassical growth theory with international trade. Most neoclassical growth models are based on the pioneering works of Solow (1956). The standard neoclassical growth theory assumes that capital and labor are substitutes for one another with the result that the long-run growth path of the economy is one of full employment. The Solow model has been extended and generalized in many studies (e.g., Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). The world economy consists of multiple countries, indexed by $j = 1, \ldots, J$. Country j has a fixed population, $\bar{N}_j$, ($j = 1, \ldots, J$). In order to describe national economies, we follow the Uzawa model by assuming that each economy has two sectors. We call them respectively tradable sector and non-tradable sector. Although the production side of our model is based on the neoclassical growth approach, we use an alternative approach to consumer behavior proposed by Zhang (1993). We assume that all the economy can produce a homogenous tradable commodity (see also Ikeda and Ono, 1992). The commodity is like the commodity in the Solow model which can be consumed and invested. Each economy can thus produce one (durable) good in the global economy and one non-tradable (national) good. Households own the assets of the economy and distribute their incomes to consume; and to save. Production sectors use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of the hoarding of output in the form of non-productive inventories held by households. We assume that the global economy has a fixed amount of gold owned by households. Gold can be sold in free markets without any friction and transaction costs. The assumption of fixed amount of gold is strict requirements (Barro, 1979; Barsky and Summers, 1988; and Chappell and Dowd, 1997). Gold mining is an important industry and new supply brings about changes in gold markets. For the simplicity of analysis, we omit these complicated issues. Let price be measured in terms of the tradable good and the price of the good be unit. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in country j. Capital depreciates at a constant exponential rate δ_j in country j, being independent of the manner of use within each country. Depreciation rates may vary between countries. Let $p_g(t)$ and $p_{js}(t)$ denote the (internationally equal) price of gold and the price of non-tradable good. We use subscript index, i and s to stand for tradable good sector and non-tradable good sector, respectively, in country j. We use $N_{jm}(t)$ and $K_{jm}(t)$ to stand for the labor force and capital stocks employed by sector m in country j. Let $F_{jm}(t)$ stand for the output level of sector m in country j.

2.1. The labor supply

The aggregated labor force $N_j(t)$ of country j is given by

$$N_j = h_j \bar{N}_j, \qquad (1)$$

where h_j is the level of human capital in country j.

2.2. Production functions

We assume that production of sector (j, q) is to combine 'qualified labor force', $N_{jq}(t)$, and physical capital, $K_{jq}(t)$. We use the conventional production function to describe the relationship between inputs and output. The production process is described by

$$F_{jq}(t) = A_{jq} K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \; A_{jq}, \alpha_{jq}, \beta_{jq} > 0, \; \alpha_{jq} + \beta_{jq} = 1, \quad (2)$$

where A_{jq}, α_{jq}, and β_{jq} are positive parameters. The production functions are neoclassical. They are homogeneous of degree one with the inputs. In this study, we assume that levels of human capital are exogenous and total factor productivities are fixed.

2.3 Marginal conditions

Each production sector chooses the two variables $K_{ji}(t)$ and $N_{ji}(t)$ to maximize its profit. The marginal conditions are

$$r(t) + \delta_j = \frac{\alpha_{ji} F_{ji}(t)}{K_{ji}(t)}, \quad w_j(t) = \frac{\beta_{ji} F_{ji}(t)}{N_{ji}(t)}, \quad (3)$$

$$r(t) + \delta_j = \frac{\alpha_{js} p_{js}(t) F_{js}(t)}{K_{js}(t)}, \quad w_j(t) = \frac{\beta_{js} p_{js}(t) F_{js}(t)}{N_{js}(t)}, \quad (4)$$

where δ_j is depreciation rate of physical capital in country j.

2.4. Choice between physical wealth and gold

This study assumes that gold is privately owned by households. Gold can be sold and bought in free markets without any friction and transaction costs. Gold use will not waste it and it cannot regenerate itself. Households can own gold and physical wealth. In order to model the cost of keeping and using gold, we assume that gold can be “rented” through markets for decoration use. We consider that the gold which is owned by the representative household can be used either by the household for decoration or rented out to other households. The rent of gold is denoted by $R_g(t)$. Consider now an investor with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in gold thereby earning a profit equal to the net own-rate of return $R_g(t)/p_g(t)$. As we assume capital and gold markets to be at competitive equilibrium at any point in time, two options must yield equal returns, i.e.

$$\frac{R_g(t)}{p_g(t)} = r(t). \quad (5)$$

This equation enables us to determine choice between owning gold and (physical) wealth. It is obvious that the assumption is made under many strict conditions. For instance, we neglect any transaction costs and any time needed for buying and selling. Expectations on gold are complicated. It should be noted that if the expected returns for the two assets in the future are equal, equation (5) may hold. Equation (5) also implies perfect information.

2.5. Consumer behavior

Consumers decide consumption levels of goods and gold, as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993). We denote respectively the representative household's physical wealth by $\bar{k}_j(t)$, the amount of gold by $\bar{g}_j(t)$. The total value of wealth owned by the household $a_j(t)$ is the sum of the two assets' values

$$a_j(t) = \bar{k}_j(t) + p_g(t)\bar{g}_j(t). \quad (6)$$

Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, the wage payment $w_j(t)$, and the gold interest income $R_g(t)\bar{g}_j(t)$ is

$$y_j(t) = r(t)\bar{k}_j(t) + h_j w_j(t) + R_g(t)\bar{g}_j(t). \quad (7)$$

We call $y_j(t)$ the current income. The per capita disposable income is given by

$$\hat{y}_j(t) = y_j(t) + a_j(t). \quad (8)$$

The disposable income is used for saving and consumption. At each point in time, the representative household would distribute the total available budget between saving $s_j(t)$, consumption of tradeable good $c_j(t)$, non-tradable good $c_{js}(t)$, and use of gold for decoration $\hat{g}_j(t)$. The budget constraint is given by

$$c_j(t) + p_{js}(t)c_{js}(t) + s_j(t) + R_g(t)\hat{g}_j(t) = \hat{y}_j(t). \quad (9)$$

The representative household has four variables, $s_j(t)$, $c_j(t)$, $c_{js}(t)$, and $\hat{g}_j(t)$, to decide. The consumer's utility function is specified as follows

$$U_j(t) = c_j^{\xi_{0j}}(t)c_{js}^{\mu_{0j}}(t)\hat{g}_j^{\gamma_{0j}}(t)s_j^{\lambda_{0j}}(t), \ \xi_{0j}, \mu_{0j}, \gamma_{0j}, \lambda_{0j} > 0,$$

in which ξ_{0j}, μ_{0j}, γ_{0j}, and λ_{0j} are the household's elasticities of utility with regard to tradable good, non-tradable good, gold decoration, and saving. We call ξ_{0j}, μ_{0j}, γ_{0j}, and λ_{0j} propensities to consume tradable good, to consume non-tradable good, to use gold, and to hold wealth, respectively.

Maximizing $U_j(t)$ subject to (9) yields

$$c_j(t) = \xi_j \hat{y}_j(t), \ p_{js}(t)c_{js}(t) = \mu_j \hat{y}_j(t), \ R_g(t)\hat{g}_j(t) = \gamma_j \hat{y}_j(t), \ s_j(t) = \lambda_j \hat{y}_j(t), \quad (10)$$

where

$$\xi_j \equiv \rho_j \xi_{0j}, \ \mu_j \equiv \rho_j \mu_{0j}, \ \gamma_j \equiv \rho_j \gamma_{0j}, \ \lambda_j \equiv \rho_j \lambda_{0j}, \ \rho_j \equiv \frac{1}{\xi_{0j} + \mu_{0j} + \gamma_{0j} + \lambda_{0j}}.$$

2.7. Wealth accumulation

According to the definition of $s_j(t)$, the change in the household's wealth is given by

$$\dot{a}_j(t) = s_j(t) - a_j(t). \quad (11)$$

The equation simply states that the change in wealth is equal to the saving minus the dissaving.

2.8. Gold owned by households

The gold owned by the population is equal to the available amounts of the asset

$$\sum_{j=1}^{J} \bar{g}_j(t)\bar{N}_j = G. \quad (12)$$

2.9. Gold being fully used for decoration

The amount of gold used for decoration by the population is equal to the total gold

$$\sum_{j=1}^{J} \hat{g}_j(t)\bar{N}_j = G. \quad (13)$$

2.10. Market clearing in non-tradable good markets

The demand for non-tradable good equals the supply at any point in time in each country

$$c_{js}(t)\bar{N}_j = F_{js}(t). \quad (14)$$

2.11. National capital stock is fully employed

The national capital stock is fully employed

$$K_{ji}(t) + K_{js}(t) = K_j(t), \quad j = 1, \ldots, J. \quad (15)$$

2.12. National physical wealth being owned by the domestic households

$$\sum_{j=1}^{J} \bar{k}_j(t)\bar{N}_j = \bar{K}_j(t), \quad j = 1, \ldots, J. \qquad (16)$$

2.13. Full employment of the labor force

We assume that the labor force is fully employed

$$N_{ji}(t) + N_{js}(t) = N_j. \qquad (17)$$

2.14. Market clearing in tradable good markets

The total capital stock in international markets employed by the production sectors is equal to the total wealth owned by all the countries. That is

$$K(t) \equiv \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t)\bar{N}_j. \qquad (18)$$

The world production is equal to the world consumption and world net savings. That is

$$C(t) + S(t) - K(t) + \sum_{j=1}^{J} \delta_j K_j(t) = F(t),$$

where

$$C(t) \equiv \sum_{j=1}^{J} c_j(t)\bar{N}_j, \quad S(t) \equiv \sum_{j=1}^{J} s_j(t)\bar{N}_j, \quad F(t) \equiv \sum_{j=1}^{J} F_{ji}(t).$$

2.15. International trade

The trade balances of the economies are given by

$$E_j(t) = \left(\bar{K}_j(t) - K_j(t)\right) r(t), \quad j = 1, \ldots, J. \quad (19)$$

When $E_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $E_j(t)$ is zero, country $j's$ trade is in balance. Equations (19) imply

$$\sum_{j=1}^{J} E_j(t) = 0.$$

We built the model with trade, economic growth, physical and gold distribution in the world economy in which the domestic markets of each country are perfectly competitive while, international product, gold and capital markets are freely mobile. The model synthesizes main ideas in economic growth theory and trade theory in a comprehensive framework. The model is built on many strict assumptions. Nevertheless, from a structural point of view the model is general in the sense that some well-known models in economics can be considered as special cases. For instance, if the countries are identical and human capital is constant, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961, 1963). Our model is also structurally similar to the Oniki-Uzawa trade model (Oniki and Uzawa, 1965). It is built on the basis of the Uzawa-Lucas two sector model.

3. THE DYNAMICS, EQUILIBRIUM AND STABILITY

The economic system contains many variables. These variables are nonlinearly related. For illustration, the rest of the study simulates the model. In order to simulate the model with computer, we provide a computational procedure so that one can easily follow the motion of the economic system with any set of parameters and initial conditions. In the appendix, we show that the dynamics of the economy can be expressed as J differential equations. First, we introduce a variable $z_1(t)$ by

$$z_1(t) \equiv \frac{r(t) + \delta_j}{w_1(t)}.$$

We now show that the dynamics can be expressed by differential equations with $z_1(t)$ and $\{a_j(t)\} = (a_2(t), \ldots, a_J(t))$ as the variables.

Lemma

The motion of J variables, $z_1(t)$, and $\{a_j(t)\}$, is given by the following J differential equations

$$\dot{z}_1(t) = \Lambda_1(z_1(t), \{a_j(t)\}),$$

$$\dot{a}_j(t) = \Lambda_j(z_1(t), \{a_j(t)\}), \quad j = 2, \ldots, J, \quad (20)$$

where $\Lambda_j(t)$ are functions of $z_1(t)$ and $\{a_j(t)\}$ defined in the appendix. The values of the other variables are given as functions of $z_1(t)$ and $\{a_j(t)\}$ at any point in time by the following procedure: $r(t)$ and $w_j(t)$ by (A2) → $z_j(t)$ by (A3) → $a_1(t)$ by (A3) → $p_{js}(t)$ by (A4) → $N_{js}(t)$ by (A10) → $N_{ji}(t)$ by (A11) → $K_{ji}(t)$ and $K_{js}(t)$ by (A1) → $K_j(t) = K_{js}(t) + K_{ji}(t)$ → $K(t)$ by (18) → $p_g(t)$ by (A8) → $R_g(t)$ by (5) → $\hat{y}_j(t)$ by (A6) → $F_{jq}(t)$ by (1) → $c_j(t)$, $c_{js}(t)$, $\hat{g}_j(t)$, and $s_j(t)$ by (10).

For simulation, we specify the values of the parameters. We consider that the world consists of three national economies, i.e., $J = 3$. We specify the parameter values as follows

$$N_1 = 10,\ N_2 = 20,\ N_3 = 40,\ G = 10,\ h_1 = 5,\ h_2 = 3,\ h_3 = 1,\ \delta_1 = 0.05,\ \delta_2 = 0.055,$$
$$\delta_3 = 0.05,\ \alpha_{1i} = 0.34,\ \alpha_{2i} = 0.32,\ \alpha_{3i} = 0.32,\ \alpha_{1s} = 0.34,\ \alpha_{2s} = 0.34,\ \alpha_{3s} = 0.34,$$
$$A_{1i} = 1.2,\ A_{2i} = 1.1,\ A_{3i} = 1,\ A_{1s} = 1.1,\ A_{2s} = 1,\ A_{3s} = 0.9,\ \lambda_{10} = 0.8,\ \mu_{10} = 0.06,$$
$$\xi_{10} = 0.06,\ \gamma_{10} = 0.02,\ \lambda_{20} = 0.75,\ \mu_{20} = 0.06,\ \xi_{20} = 0.06,\ \gamma_{20} = 0.015,$$
$$\lambda_{30} = 0.7,\ \mu_{30} = 0.06,\ \xi_{30} = 0.06,\ \gamma_{30} = 0.01,$$

Country 1, 2 and 3's populations are respectively 10, 20 and 40. Country 3 has the largest population. Country 1 has the highest human capital and Country 2 is next. The physical capital depreciation rates of the three economies are approximately 0.05. The total factor productivities are different between three economies. Country 1's total factor productivity is highest and Country 3's total factor productivity is lowest. The output elasticities with respect to labor and capital also vary between countries. We specify the values of the parameters, α_{ji} and α_{js} in the Cobb-Douglas productions approximately equal to 0.3. The household preferences of the three economies also vary. As we already provided the procedure to follow the motion of each variable in the system, it is straightforward to plot the motion with computer. We specify the initial conditions as follows

$$z_1(0) = 0.08, \quad a_2(0) = 29, \quad a_3(0) = 9.3.$$

The motion of the system is given in Figure 1. In the figure, the GDPs $Y_j(t)$ are defined as follows

$$Y_j \equiv F_{ji} + p_{js} F_{js}.$$

Because of the chosen initial values, the global wealth and all the GDP are enhanced over time. The rate of interest rises in association with the raise in the wage rates. The prices of the non-tradable goods fall over time. The price and rent

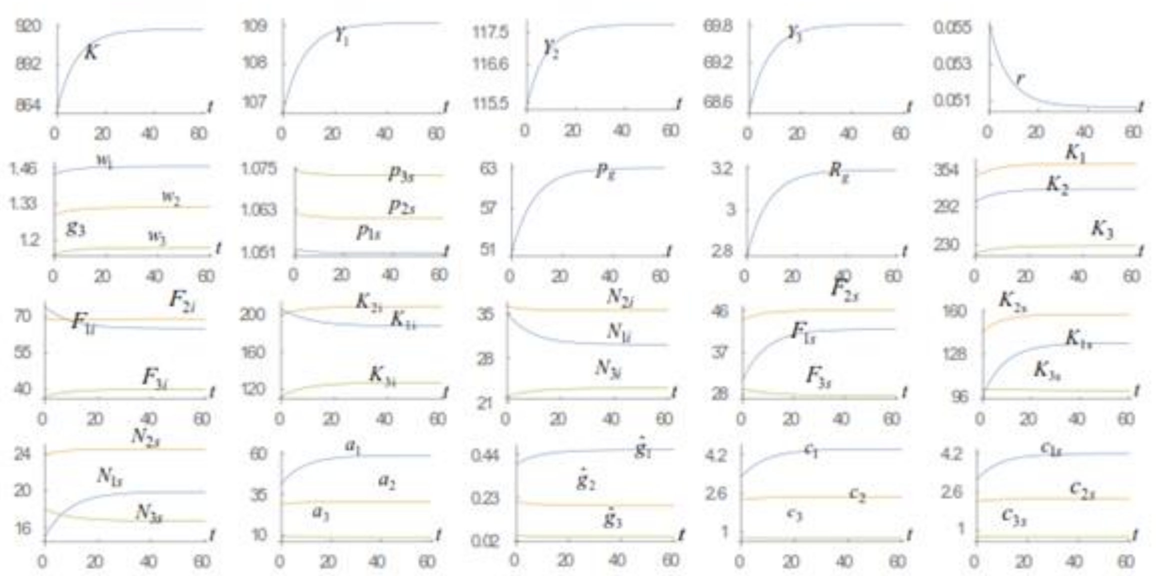

Figure 3.1. The Motion of the Global Economy

of gold are increased. The capital stocks employed by the three economies are increased. The output level of country 1's tradable sector is reduced and the output level of the non-tradable sector is increased. Country 1's tradable sector employs less capital stock and labor force while the country's non-tradable sector employs more capital good and labor force. The output levels of country 2's two sectors are slightly increased. Country 2's tradable sector employs more capital stock and more labor force, and the country's non-tradable sector employs more capital good and less labor force. The output level of country 3's tradable sector is increased and the output level of the non-tradable sector is reduced. Country 3's tradable sector employs more capital stock and labor force and the non-tradable sector employs less capital good and labor force. The representative household in Country owns more wealth, consumes the two goods more and uses more gold. The representative households' wealth levels, consumption levels of tow goods, and amount of gold-use in the other two economies are slightly changed.

As shown in Figure 1, different countries will not experience convergence in per capita income, consumption and wealth in the long term as they are different in preferences and total productivities. There are extensive discussions about income and wealth convergence between nations in the literature of economic growth and development. The literature provides little insights into the issues as most of these studies are based on the insights from analyzing models of closed economies (Barro and Sala-i-Martin, 1995). As economics lacks analytical frameworks for analyzing global growth and trades with microeconomic foundation, theoretical economics fails to discuss issues related to global income and wealth convergence. For instance, the conclusions from the Solow model for closed economies are often used to discuss issues related to income inequalities between countries. The Solow model predicts that convergence in income levels among closed countries is achieved by faster accumulation of physical capital in the poor countries. In a recent empirical study on the determinants of economic growth and investment with a panel of around 100 countries from 1960 to 1995, Barro (2013: 327) observes that "The data reveal a pattern of conditional convergence in the sense that the growth rate of per capita GDP is inversely related to the starting level of per capita GDP, holding fixed measures of government policies and institutions, initial stocks of human capital, and the character of the national population. With respect to education, growth is positively related to the starting level of average years of school attainment of adult males at the secondary and higher levels." Our model shows different patterns. It should be noted that we treat human capital exogenous.

From Figure 1 we observe that the system becomes stationary in the long term. Following Lemma 1 under (15), we calculate the equilibrium values of the variables as follows

$$K = 917.3, \quad r = 0.051, \quad p_g = 63.03, \quad R_g = 3.19,$$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.47 \\ 1.32 \\ 1.17 \end{pmatrix}, \quad \begin{pmatrix} p_{1s} \\ p_{2s} \\ p_{3s} \end{pmatrix} = \begin{pmatrix} 1.05 \\ 1.06 \\ 0.07 \end{pmatrix}, \quad \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 109.1 \\ 117.7 \\ 69.8 \end{pmatrix}, \quad \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 323.5 \\ 365.9 \\ 227.9 \end{pmatrix}, \quad \begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 64.9 \\ 68.8 \\ 40.1 \end{pmatrix},$$

$$\begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 30.1 \\ 35.5 \\ 23.3 \end{pmatrix}, \quad \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 187.6 \\ 208.5 \\ 127.4 \end{pmatrix}, \quad \begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 42.1 \\ 46.1 \\ 27.7 \end{pmatrix}, \quad \begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 19.9 \\ 24.5 \\ 16.8 \end{pmatrix},$$

$$\begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 135.8 \\ 157.4 \\ 100.5 \end{pmatrix}, \quad \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 58.9 \\ 30.6 \\ 8.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{g}_1 \\ \bar{g}_2 \\ \bar{g}_3 \end{pmatrix} = \begin{pmatrix} 0.46 \\ 0.19 \\ 0.04 \end{pmatrix}, \quad \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 4.42 \\ 2.45 \\ 0.74 \end{pmatrix}, \quad \begin{pmatrix} c_{1s} \\ c_{2s} \\ c_{3s} \end{pmatrix} = \begin{pmatrix} 4.21 \\ 2.31 \\ 0.69 \end{pmatrix}.$$

It is straightforward to calculate the three eigenvalues as follows

$$\{-0.124, \ -0.112, \ -0.107\}.$$

This implies that the world economy is stable. Hence, we can effectively conduct comparative dynamic analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

It is important to ask questions such as how a change in one country's conditions affects the national economy and global economies. For instance, if a country changes its preference to use gold, how the global gold market and other economic variables are affected over time. We can easily answer the question as we can simulate the motion of the dynamic system. This section examines effects of changes in some parameters on the global economy. First, we introduce a variable $\Delta x(t)$ to stand for the change rate of the variable the parameter value.

4.1. Country 1 enhancing its human capital

We now show effects of the following change in country 1's human capital: h1 :5 ⍰ 5.5. The simulation result is plotted in Figure 2. As the system contains many variables and these variables are connected to each other in nonlinear relations, it is difficult to verbally explain these relations over time. As the human capital is increased, Country 1's total labor supply is increased. The total capital and capital stock employed by the country are increased and the capital stocks employed by the other two countries are slightly affected. The wage rates of the three economies are slightly reduced in association with rising rate of interest. The representative household of Country 1 has more wealth, uses more gold and consumes two goods more. The representative households in the other two countries use less gold and their wealth and consumption are slightly affected. The price and rent of gold are increased. The prices of the non-tradable goods are slightly increased. Country 1's GDP is increased and the other two countries' GDPs are reduced. Country 1 increases its two sectors' inputs and output levels. Country 2 (Country 3) increases the non-tradable sector output and reduces the tradable sector output. Country 2 (Country 3)increases the two sectors' capital inputs and the non-trade sector's labor input.

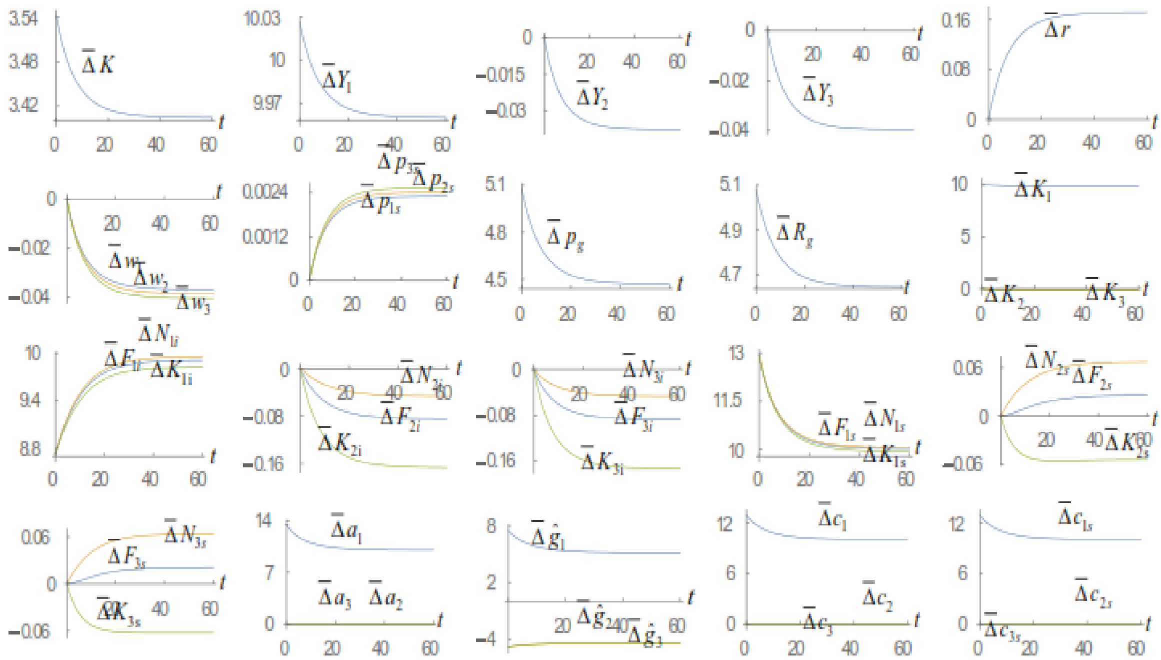

Figure 4.1.1. Country 1 Enhancing Its Human Capital

4. 2. A rise in the total factor productivity of country 1's trade sector

We study the effects of a rise in country 1's tradable sector on the global economy. It has been argued that productivity differences explain much of the variation in incomes across countries, and technology plays a key role in determining productivity. We see what will happen to the global economy when A_{1i} : 1.2 ⇒ 1.3. The simulation result is plotted in Figure 3. As the technology is improved, Country 1's tradable sector increases its output and capital input. Its labor input is increased initially and is not affected in the long term. The wage rate in this country is increased and the wage rates in the other two economies are reduced. The gold price and rent are reduced initially and increased in the long term. Country 1's representative household owns less wealth, uses less gold and consumes less the two goods initially and owns more wealth, uses more gold and consumes the two goods more in the long term. It should be noted that except the amounts of gold that are consumed by Countries 2 and 3, in the long term the GDPs, the output levels, inputs and the households' wealth and consumption levels in the other countries arealmost not affected by the technological improvement in Country 1.

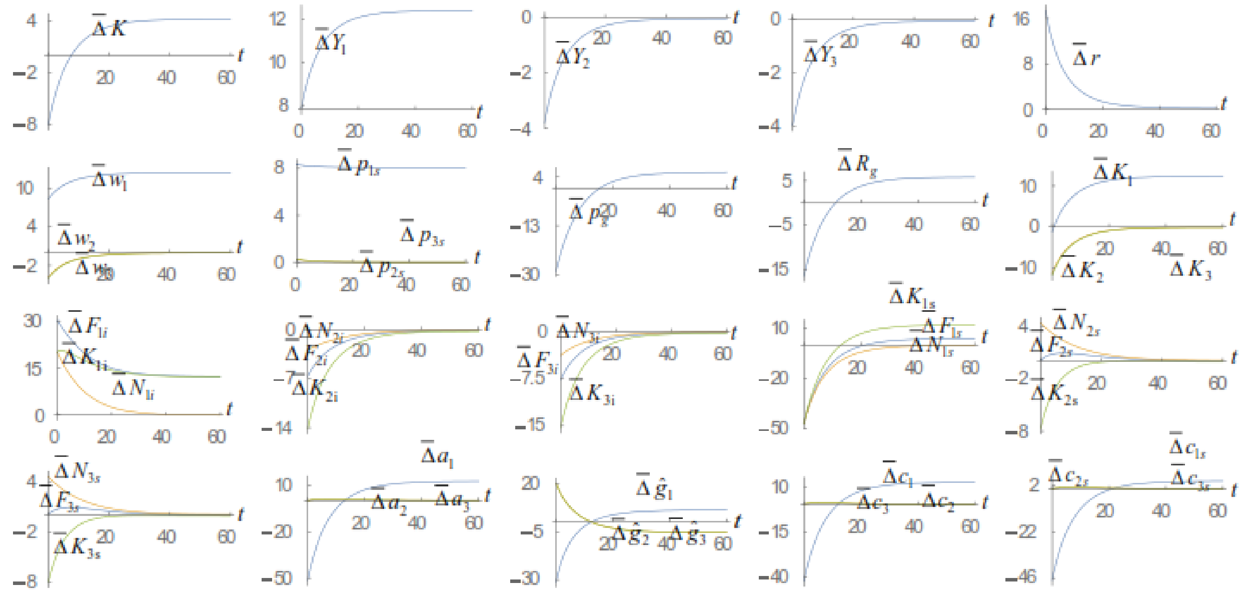

Figure 4.2.1. A Rise in the Total Factor Productivity of Country 1's Trade Sector

4.3. A rise in country 1's population

It is held observed that the effect of population growth on economic growth varies with the level of economic development and can be either positive or negative for different economies (with regard to different economic indicators). Although this study assumes exogenous population, it is important to examine effects of exogenous change in the population. As our

study does not allow immigration, our conclusions are limited for the contemporary global economy. We now increase Country 3's population in the following way: $N_3 : 40 \Rightarrow 45$. The simulation results are plotted in Figure 4. The global wealth is increased by the population expansion in Country 3. Country 1's GDP falls slightly initially and is increased in the long term. Countries 2 and 3's GDPs are increased. The wage rates are slightly increased in association with the falling rate of interest. The gold price and rent are increased in the global market. The gold use by each consumer is reduced. In the long term, the wealth levels and consumption levels are almost not affected. As far as the per capita wealth and consumption of goods are concerned, in the long term the change in Country 3's population has almost no impact on the global economy, even though the change affects the fixed resource (gold) distribution. In this study the main growth mechanism is wealth accumulation in a free global economy with constant returns to scale. If we take account of returns to scale effects or introduce resources such as land and energies which are necessary for production, changes in the population may have long-term effects on the global economy in terms of per capita wealth and consumption.

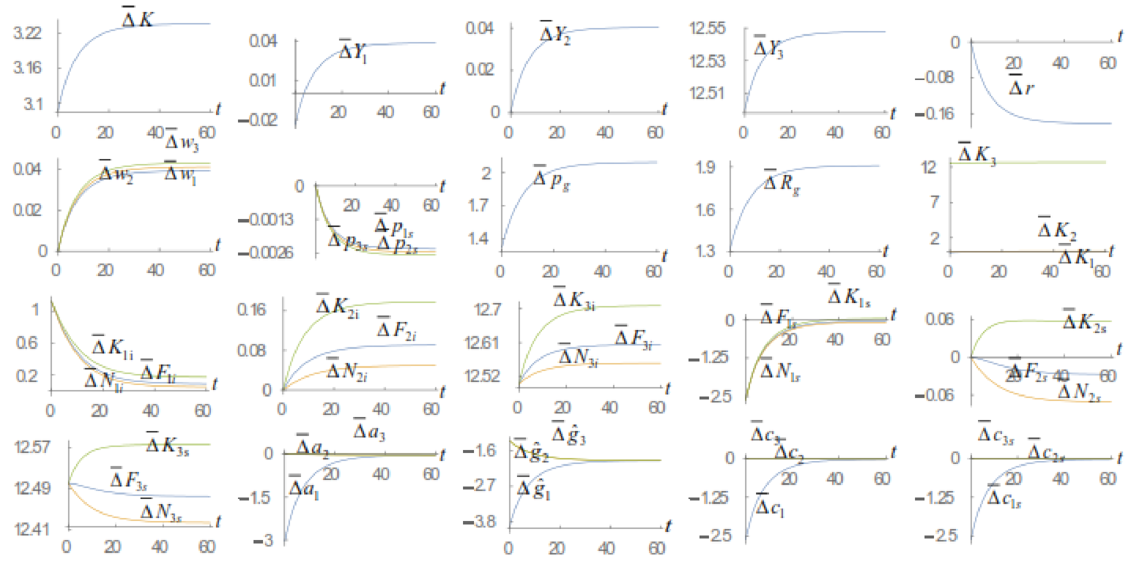

Figure 4.3.1. A Rise in Country 1's Population

4.4. Country 1 increasing the propensity to use gold

We now show effects of the following change in Country 1's propensity to use gold: γ_{10} :0.02 $\Rightarrow$ 0.025. The simulation result is plotted in Figure 5. The household in Country 1 appreciates gold more, for instance, as a symbol of social status, the gold price and rents are increased. The household in Country 1 consumes gold more. The household in Country 1 initially owns more wealth and consumes the two goods more and in the long term owns less wealth and consumes less the two goods. The consumers in the other two economies use less gold, own more wealth, and consume the two goods more. The global wealth is reduced. The GDPs of the three economies are all reduced. Each country uses less capital inputs. The rate of interest is increased in association with rising in wage rates. The economic structural change is illustrated in the figure.

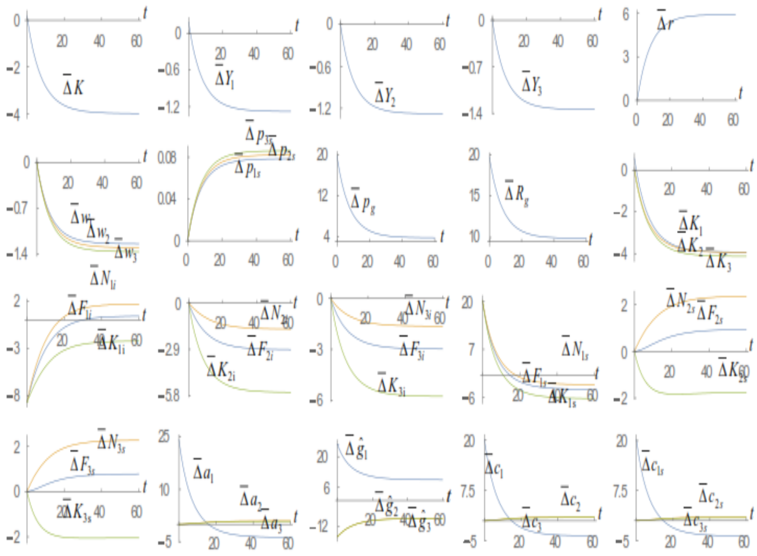

Figure 4.4.1. Country 1 Increasing the Propensity to Use Gold

4.5. Country 1 increasing the propensity to consume non-tradable good

We now study what will happen to the global economy when Country 1 increases the propensity to consume its domestic non-tradable product. We allow the propensity to consume the non- tradable good as follows: μ_{10} : 0.06 $\Rightarrow$ 0.07.The simulation result is plotted in

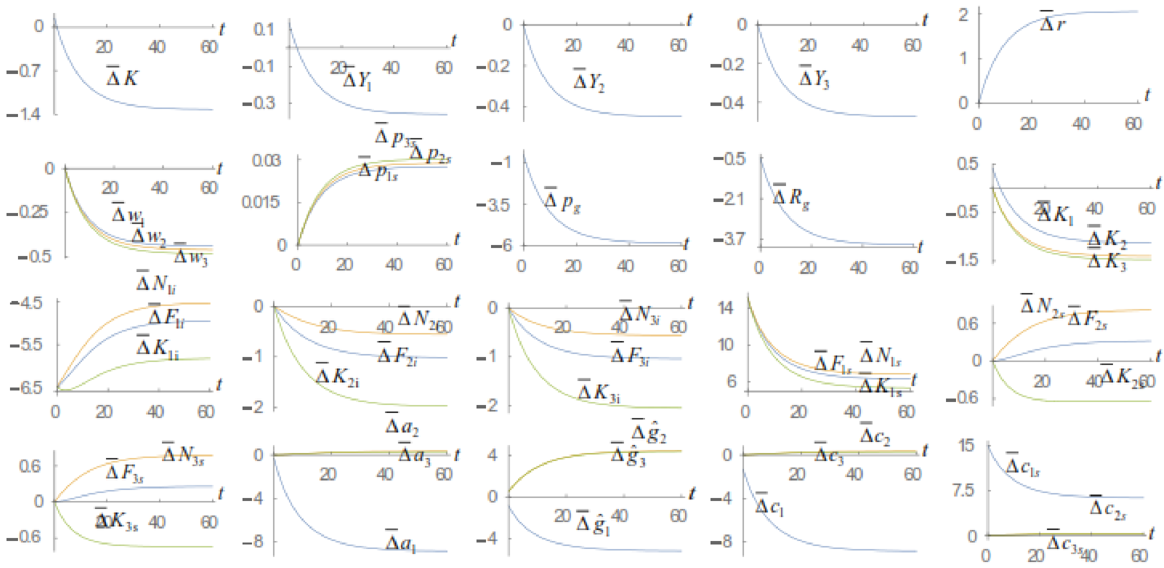

Figure 4.5.1. Country 1 Increasing the Propensity to Consume Non-Tradable Good

Figure 6. As the preference is changed, the representative household of Country 1 holds less wealth, uses less gold, consumes the non-tradable good more and the tradable good less. The households in the other two economies use gold more, have wealth a little more, and consume a little more the two goods. The global wealth falls. Country's GDP rises initially and falls in the long term. The other two economies' GDPs fall. The rate of interest rises. The wage rates are reduced and the prices of the non-tradable goods are slightly enhanced. The gold price and rent are reduced.

4.6. Country 3's propensity to save being augmented

We now increase Country 3's propensity to save in the following way: λ_{30} : 0.7 $\Rightarrow$ 0.75. The simulation results are plotted in Figure 7. The global capital and all the countries' GDPs are increased. Country 1 uses less gold. Country 2 uses gold more initially and less in the long term. Country 3 uses gold less initially and more in the long term. The gold price and rent are reduced initially and increased in the long term. All the economies employ more capital inputs. The rate of interest and prices of all the non-tradable goods are reduced. The wage rates are increased.

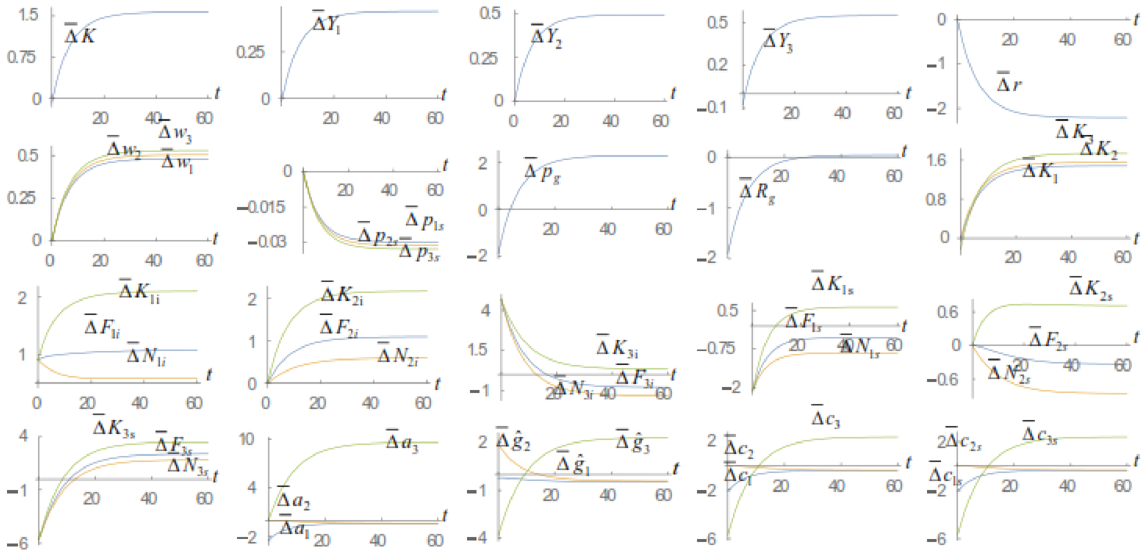

Figure 4.6.1. Country 3's Propensity to Save Being Augmented

5. CONCLUDING REMARKS

This study deals with dynamics of gold price in global markets. We introduced gold into a general dynamic equilibrium growth model with multiple countries and free trades between countries. The model was developed by integrating the Solow growth model, the Uzawa two-sector growth model, and the Oniki–Uzawa trade model within a comprehensive framework. The model synthesized these well-known economic models with Zhang's utility function to determine household behavior. It is built for any number of national economies. Each national economy consists of one tradable and one non-tradable sectors. The model describes a dynamic interdependence among wealth accumulation, and division of labor, gold-use distribution, and wealth and capital distribution under perfect competition. National economies are different in population, technologies, propensities to save, propensities to use gold, and propensities to consume. We demonstrated that the dynamics of the J-country world economy can be described by J differential equations. We simulated the model, demonstrating the existence of a unique equilibrium point, describing the motion of the dynamic system, and showing (local) stability of the equilibrium point. We also demonstrated how the changes in the propensity to use gold, the population, the propensity to save, and the total factor productivities affect global economic development. Our comparative dynamic analyses provided some important insights into interactions between global economic growth and resource distributions. It should be remarked that the economic structures and interactions in our model are delicately interrelated. Our comparative dynamic analysis is limited to a few cases. We might get more insights from further simulation. We may extend the model in some directions. We may introduce some kind of government intervention into the model. The Solow model, the Uzawa two-sector growth, and the Oniki-Uzawa trade model are most well-known models in the literature of growth theory. Many limitations of our model and possible extensions and generalizations become apparent in the light of the sophistication of the literature.

6. APPENDIX

By (3) we obtain

$$z_j \equiv \frac{r + \delta_j}{w_j} = \frac{N_{jq}}{\bar{\beta}_{jq} K_{jq}}, \qquad \text{(A1)}$$

where

$$\bar{\beta}_{jq} \equiv \frac{\beta_{jq}}{\alpha_{jq}}.$$

From (A1) and (3), we obtain

$$r(z_1) = \alpha_j z_j^{\beta_{jq}} - \delta_j, \qquad \text{(A2)}$$

where

$$\alpha_j \equiv \alpha_{ji} A_{ji} \bar{\beta}_{ji}^{\beta_{ji}}.$$

From (A2) we also have

$$z_j(z_1) = \left(\frac{r + \delta_j}{\alpha_j}\right)^{1/\beta_{ji}}. \qquad \text{(A3)}$$

From (A1) we have

$$w_j(z_1) = \frac{r + \delta_j}{z_j}.$$

From (4) we have

$$p_{js}(z_1) = \frac{\bar{\beta}_{js}^{\alpha_{js}} w_j z_j^{\alpha_{js}}}{\beta_{js} A_{js}}. \qquad \text{(A4)}$$

From (6)-(8)

$$\hat{y}_j = (1 + r)\bar{k}_j + h_j w_j + (R_g + p_g)\bar{g}_j. \qquad \text{(A5)}$$

Insert (5) in (A5)

$$\hat{y}_j = (1 + r)a_j + h_j w_j. \qquad \text{(A6)}$$

From (A6) and (10)

$$p_g r \hat{g}_j = (1 + r)\gamma_j a_j + \gamma_j h_j w_j, \qquad \text{(A7)}$$

where we also use (5). Multiplying the two sides of (A7) with N_j and then adding the resulted equations, we have

$$p_g = \left(\sum_{j=1}^{J} R_j a_j + W\right)\frac{1}{G}, \qquad \text{(A8)}$$

where

$$R_j \equiv \left(\frac{1}{r} + 1\right)\bar{N}_j \gamma_j, \quad W(z_1) \equiv \frac{1}{r}\sum_{j=1}^{J} \gamma_j h_j \bar{N}_j w_j.$$

From $p_{js} c_{js} = \mu_j \hat{y}_j$ and (14)

$$N_{js} = \frac{\mu_j \hat{y}_j \bar{N}_j}{p_{js} f_{js}}, \qquad \text{(A9)}$$

where $f_{js} = F_{js} / N_{js}$. Insert (A6) in (A9)

$$N_{js} = n_{0j} + n_j a_j, \qquad \text{(A10)}$$

where

$$n_j(z_1) \equiv \frac{(1 + r)\mu_j \bar{N}_j}{p_{js} f_{js}}, \quad n_{0j}(z_1) \equiv \frac{h_j \mu_j \bar{N}_j w_j}{p_{js} f_{js}}.$$

From (17) and (A10)

$$N_{ji} = N_j - n_{0j} - n_j a_j. \qquad \text{(A11)}$$

Multiplying (12) with p_g and adding (18) to the resulted equation, we have

$$p_g G + \sum_{j=1}^{J} K_j = \sum_{j=1}^{J} a_j \bar{N}_j . \qquad (A12)$$

Insert (15) and then (A1) in (A12)

$$p_g G + \sum_{j=1}^{J} \left(\frac{N_{ji}}{\bar{\beta}_{ji}} + \frac{N_{js}}{\bar{\beta}_{js}} \right) \frac{1}{z_j} = \sum_{j=1}^{J} a_j \bar{N}_j . \qquad (A13)$$

Insert (A8) in (A13)

$$\sum_{j=1}^{J} \left(\frac{N_{ji}}{\bar{\beta}_{ji}} + \frac{N_{js}}{\bar{\beta}_{js}} \right) \frac{1}{z_j} = \sum_{j=1}^{J} \left(\bar{N}_j - R_j \right) a_j - W. \quad (A14)$$

Insert (A10) and (A11) in (A14)

$$\sum_{j=1}^{J} \left(\bar{N}_j - R_j - \frac{n_j \bar{\beta}_j}{z_j} \right) a_j = W + W_0 , (A15)$$

where

$$W_0(z_1) \equiv \sum_{j=1}^{J} \left(\frac{N_j}{\bar{\beta}_{ji} z_j} + \frac{\bar{\beta}_j n_{0j}}{z_j} \right), \quad \bar{\beta}_j \equiv \frac{1}{\bar{\beta}_{js}} - \frac{1}{\bar{\beta}_{ji}} .$$

Solve (A15) with a_1 as the variable

$$a_1 = \Lambda\left(z_1, \{a_j\}\right) \equiv \left[W + W_0 - \sum_{j=2}^{J} \left(\bar{N}_j - R_j - \frac{n_j \bar{\beta}_j}{z_j} \right) a_j \right] \left(\bar{N}_1 - R_1 - \frac{n_j \bar{\beta}_1}{z_1} \right)^{-1},$$

(A16)

where $\{a_j\} = (a_2, ..., a_J)$.

It is straightforward to check that all the variables can be expressed as functions of z_1 and $\{a_j\}$ at any point in time as follows: r and w_j by (A2) → z_j by (A3) → a_1 by (A3) → p_{js} by (A4) → N_{js} by (A10) → N_{ji} by (A11) → K_{ji} and K_{js} by (A1) → $K_j = K_{js} + K_{ji}$ → K by (18) → p_g by (A8) → R_g by (5) → $\hat{y}_j$ by (A6) → F_{jq} by (1) → c_j, c_{js}, $\hat{g}_j$, and s_j by (10). From this procedure and (11), we have

$$\dot{a}_1 = \Lambda_0\left(z_1, \{a_j\}\right) \equiv s_1 - a_1 , (A17)$$

$$\dot{a}_j = \Lambda_j\left(z_1, \{a_j\}\right) \equiv s_j - a_j , \quad j = 2, ..., J . (A18)$$

Taking derivatives of (A16) with respect to t yields

$$\dot{a}_1 = \frac{\partial \Lambda}{\partial z_1} \dot{z}_1 + \sum_{j=2}^{J} \Lambda_j \frac{\partial \Lambda}{\partial a_j} . \qquad (A19)$$

From (A17) and (A19), we have

$$\dot{z}_1 = \Lambda_1\left(z_1, \{a_j\}\right) \equiv \left(\Lambda_0 - \sum_{j=2}^{J} \Lambda_j \frac{\partial \Lambda}{\partial a_j} \right) \left(\frac{\partial \Lambda}{\partial z_1} \right)^{-1} . (A20)$$

We determine the motion of the system with (A18) and (A20) and the remaining variables by the procedure provided before. In summary, we proved the lemma.

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